UNIVERSITI SAINS MALAYSIA

APEX™

Entropy Stable Residual Distribution Method

Scalar Equation

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Scalar Results
Euler Results

Here are the scalar signal distribution,

$$\begin{cases} \phi_i^{\text{iso}} - \frac{1}{2}\alpha[v]_{ji} - \frac{1}{2}\beta[v]_{kj} - \frac{1}{2}\gamma[v]_{ik} = \tilde{\phi}_i \\ \phi_j^{\text{iso}} - \frac{1}{2}\alpha[v]_{kj} - \frac{1}{2}\beta[v]_{ik} - \frac{1}{2}\gamma[v]_{ji} = \tilde{\phi}_j \\ \phi_k^{\text{iso}} - \frac{1}{2}\alpha[v]_{ik} - \frac{1}{2}\beta[v]_{ji} - \frac{1}{2}\gamma[v]_{kj} = \tilde{\phi}_k \end{cases} \tag{1}$$

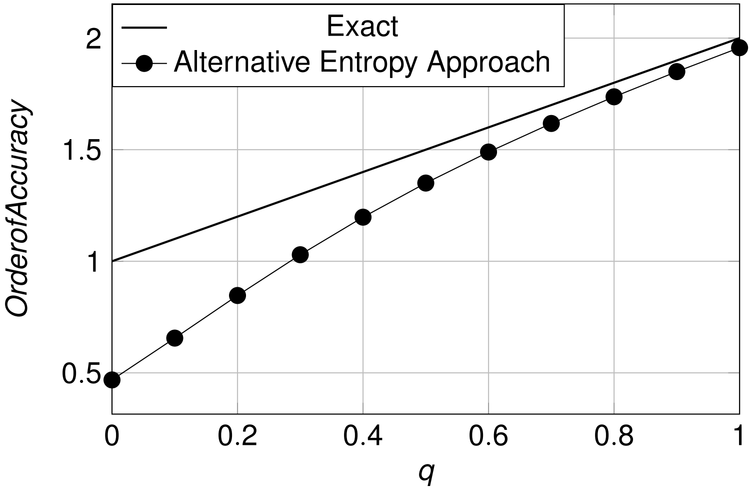
$$\alpha \geq 0 \qquad , \gamma = -\alpha, \qquad \forall \beta (\beta = 0) \tag{2}$$

$$\begin{cases} \phi_i^{\text{art}} = -\frac{1}{2}\alpha\left([v]_{ji} - [v]_{ik}\right) \\ \phi_j^{\text{art}} = -\frac{1}{2}\alpha\left([v]_{kj} - [v]_{ji}\right) \\ \phi_k^{\text{art}} = -\frac{1}{2}\alpha\left([v]_{ik} - [v]_{kj}\right) \end{cases} \tag{3}$$

Linear 2D Advection
Order of Accuracy

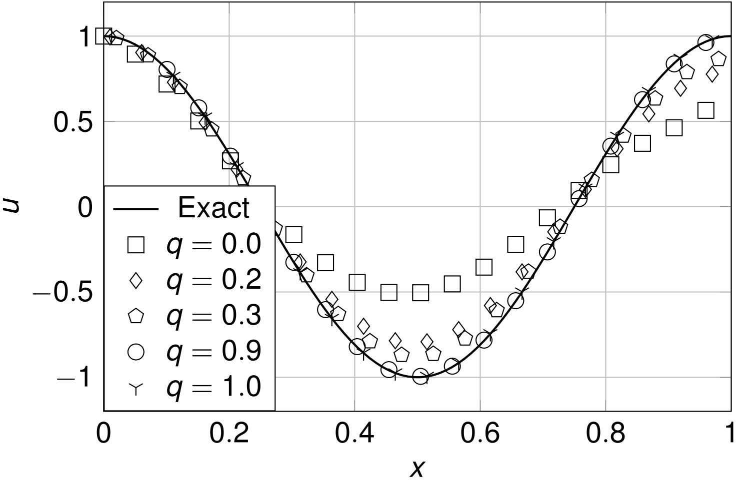
Scalar Results

Euler Results

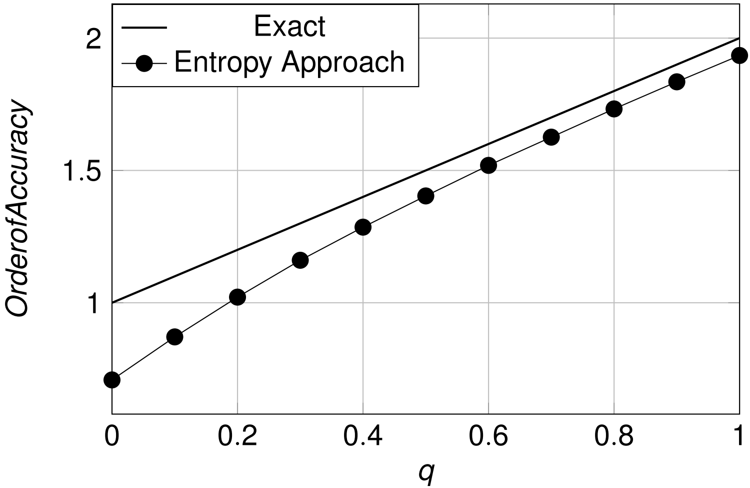


Linear 2D Advection

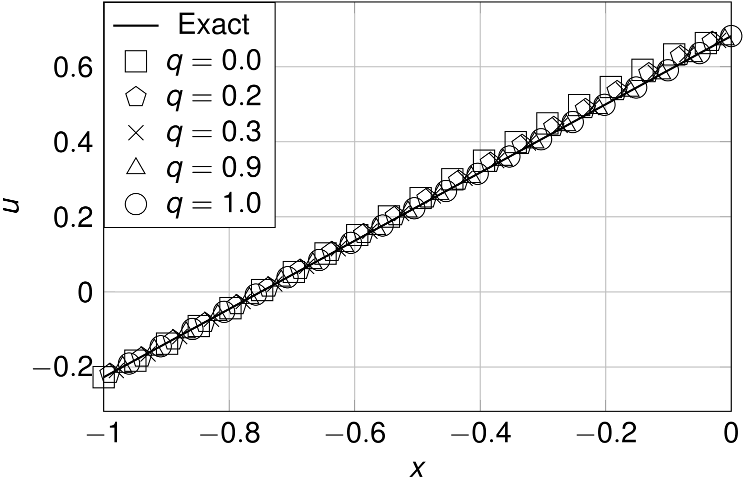
Cross Section



Linear 2D Burgers
Order of Accuracy



Linear 2D Burgers
Order of Accuracy



Here is the Euler signal distribution,

$$\phi_i = \underbrace{\frac{1}{2}\left(\vec{\boldsymbol{f}}_i - \vec{\boldsymbol{f}}^C\right)\cdot \vec{n}_i}_{\phi_i^{\text{iso}}} \underbrace{- \hat{\boldsymbol{R}}\hat{\boldsymbol{D}}_\alpha \hat{\boldsymbol{R}}^T [\boldsymbol{v}]_{ji} - \hat{\boldsymbol{R}}\hat{\boldsymbol{D}}_\beta \hat{\boldsymbol{R}}^T [\boldsymbol{v}]_{kj} - \hat{\boldsymbol{R}}\hat{\boldsymbol{D}}_\gamma \hat{\boldsymbol{R}}^T [\boldsymbol{v}]_{ik}}_{\phi_i^{\text{art}}}$$

$$\hat{\boldsymbol{D}}_\alpha = \left(\alpha|\hat{\boldsymbol{\Lambda}}|\hat{\boldsymbol{S}}\right) h, \quad \hat{\boldsymbol{D}}_\beta = \left(\beta|\hat{\boldsymbol{\Lambda}}|\hat{\boldsymbol{S}}\right) h, \quad \hat{\boldsymbol{D}}_\gamma = \left(\gamma|\hat{\boldsymbol{\Lambda}}|\hat{\boldsymbol{S}}\right) h$$

One could just amplify all waves by 1 and choose the α as follows,

$$\boldsymbol{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

This combination will be adequate for entropy-stability. Nonetheless, because all the physical waves are amplified in the same way, solution might be diffusive.

From the Taylor series analysis of the isotropic signals and the artificial terms on the linear advection, the following results are obtained. We introduce,

$$\boldsymbol{\alpha} = \begin{bmatrix} \left(\frac{h}{L_r}\right)^q & 0 & 0 & 0 \\ 0 & \left(\frac{h}{L_r}\right)^q & 0 & 0 \\ 0 & 0 & \left(\frac{h}{L_r}\right)^q & 0 \\ 0 & 0 & 0 & \left(\frac{h}{L_r}\right)^q \end{bmatrix} \tag{5}$$

Base on this α, all the waves are amplified based on a grid size (h) which is non-dimensionalized with the reference length (L_r) such as chord for the airfoil case.

To improve the robustness of the entropy-stable method without compromising the quality of the acoustics and shear waves, we propose adding more entropy generation to the entropy wave.

$$\boldsymbol{\alpha} = \begin{bmatrix} \left(\frac{h}{L_r}\right)^q & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \left(\frac{h}{L_r}\right)^q & 0 \\ 0 & 0 & 0 & \left(\frac{h}{L_r}\right)^q \end{bmatrix} \tag{6}$$

Preserving the entropy wave makes the solution more robust and stable.

Sensor
Formulation

Basic formulation,

$$\alpha_{1,3,4}^{\text{art,lim}} = \psi(S)\alpha_{1,3,4}^{\text{art,low}} + (1 - \psi(S))\alpha_{1,3,4}^{\text{art,high}} \tag{7}$$

Slope finding,

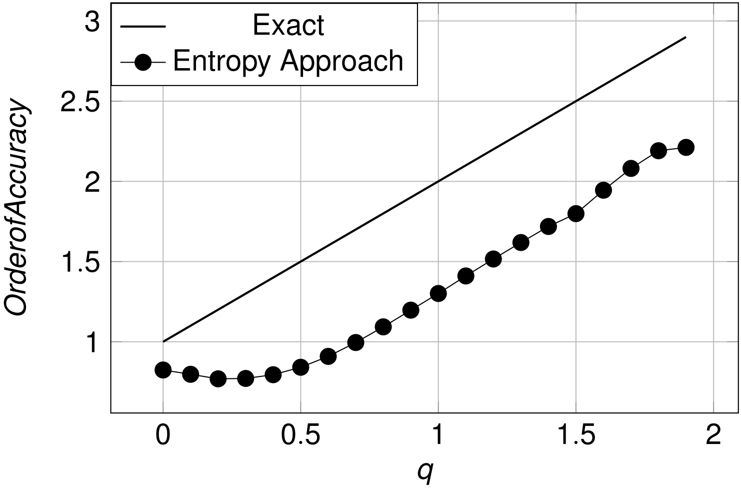
$$m_{jk}^{S} = \left| \frac{S_j - S_k}{l_i} \right| \tag{8}$$

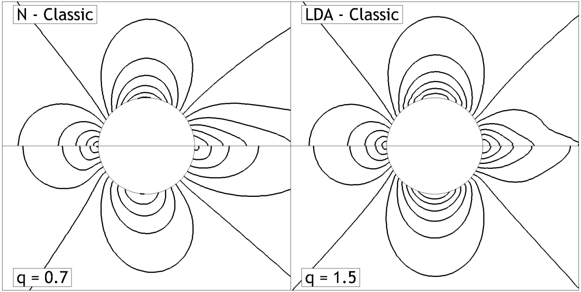
Determine the maximum slope,

$$m_{\text{max}}^{S} = \max(m_{\text{all edges}}^{S}) \tag{9}$$

Ringleb Flow

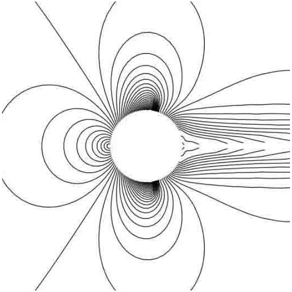
Order of Accuracy



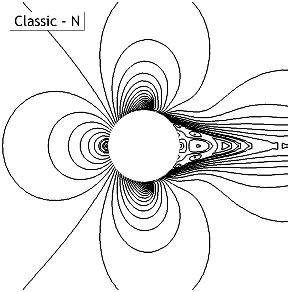


(a) N, q=0.7 (b) LDA, q=1.5

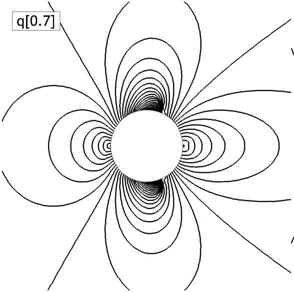
Cylinder
Transonic



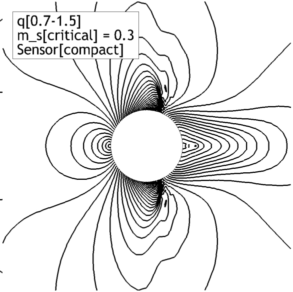
(c) Hafez



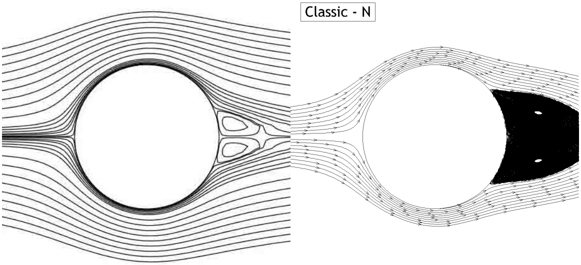
(d) Classic N



(e) q=0.7

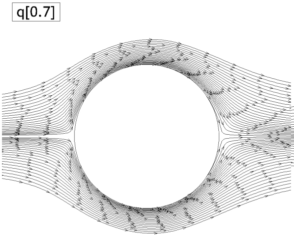


(f) q=0.7-1.5 m=0.3 S=0

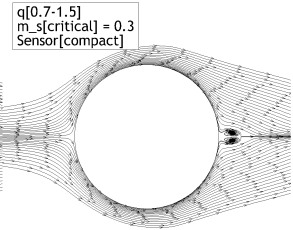


(g) Hafez (h) Classic N

Cylinder
Transonic



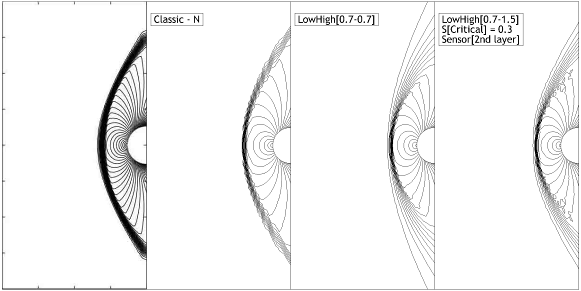
(i) q=0.7



(j) q=0.7-1.5 m=0.3 S=0

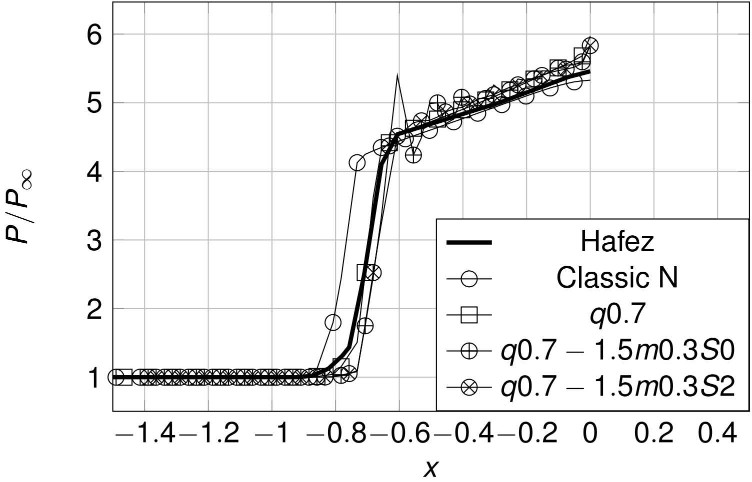
Cylinder

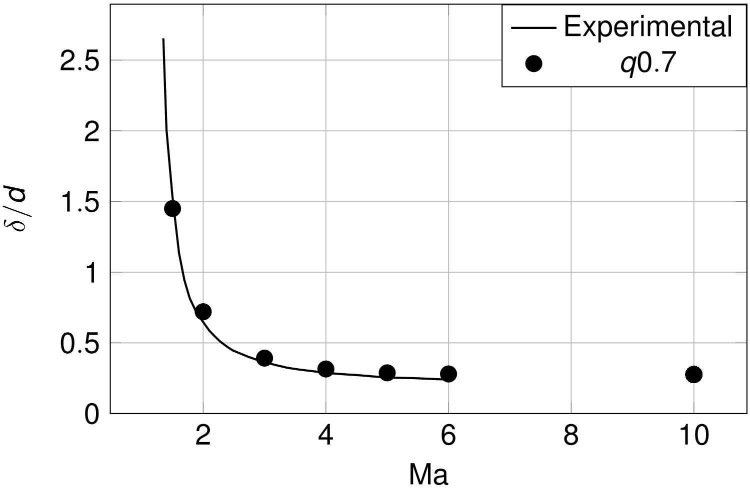
Supersonic



(k) Hafez (l) Classic N (m) q=0.7 (n) q=0.7-1.5 m=0.3 s=2

Cylinder
Supersonic




Experimental
q0.7
2.5
2
1.5
1
0.5
0
δ/d
2
4
6
8
10
Ma

Thank You

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